

FINAL EXAM (LENSTRA) - ANSWER KEY

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(1)

$$\mathbf{x}(t) = e^{-3t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

(2) (a) You really just do this by guessing! Start with 1 and x , and modify your guess!

$$y_1(x) = 1, \quad y_2(x) = x \quad y_3(x) = \frac{1}{16}e^{4x}$$

(b) Yes! The Wronskian is identically zero, hence nonzero at *some* point, and that is enough to determine linear independence!

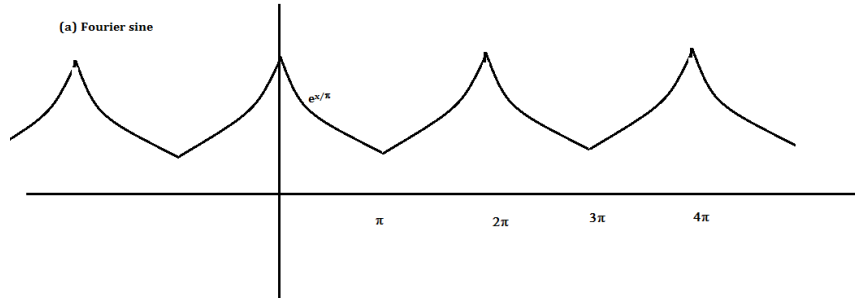
(3)

$$(D + 1)(D^2 + 4)[y] = 0$$

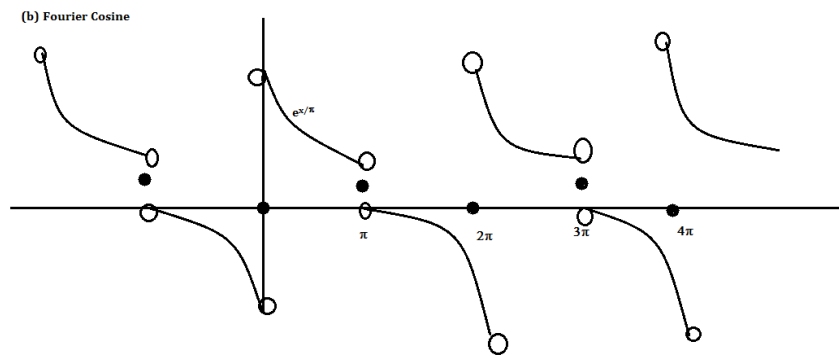
I found this because e^{-x} corresponds to a root $r = -1$ and $\cos(2x)$ corresponds to a root $r = 2i$. Hence I looked for a simple equation which has roots $-1, \pm 2i$.

$$y(t) = Ae^{-t} + B \cos(2t) + C \sin(2t)$$

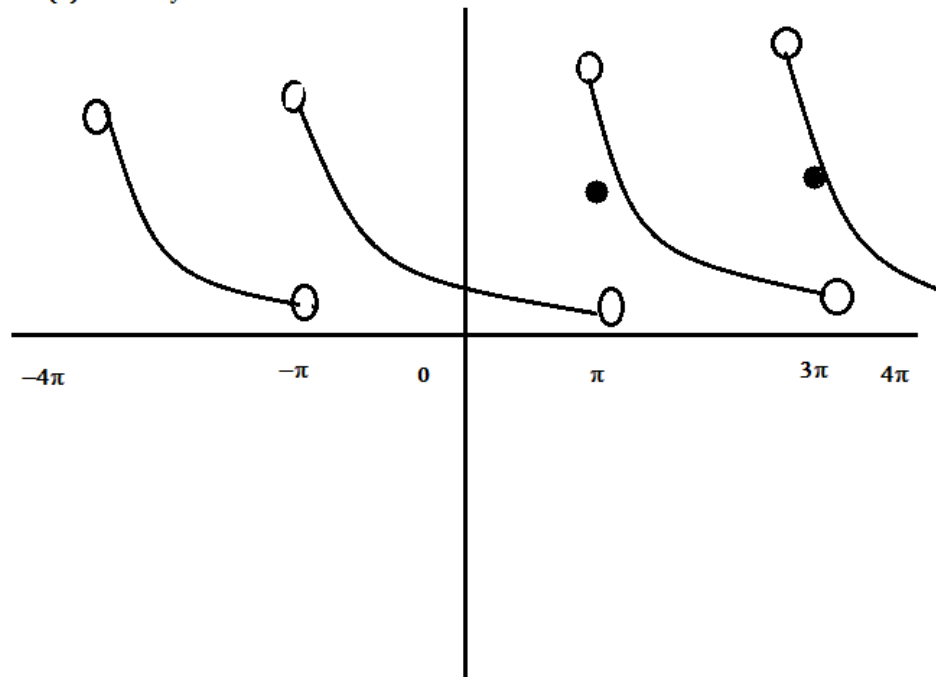
(4) 54/Practice Exams/Lenstragraph1.png



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(c) Ordinary Fourier

(5)

$$u(x, t) = \cosh(3t) \sin(3x)$$